Banking competition, collateral constraints and optimal monetary policy*

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Abstract

We analyze optimal monetary policy in a model where borrowing is subject to collateral constraints and credit flows are intermediated by a monopolistically competitive banking sector. We show that, under certain conditions and up to a second order approximation, welfare maximization is equivalent to stabilization of four goals: inflation, output gap, the consumption gap between constrained and unconstrained consumers, and the distribution of the collateralizable asset between both groups. Following both productivity and credit-crunch shocks, the optimal monetary policy commitment implies a short-run trade-off between these goals. Finally, such trade-offs become amplified as banking competition increases, due to the increase in financial leveraging.

1 Introduction

In this paper we provide a theoretical framework for the analysis of the optimal conduct of monetary policy in the presence of financial frictions. Both optimal monetary policy and the macroeconomic effects of financial frictions have attracted much attention in recent times.

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However, much less effort has been devoted to exploring the connections between both fields in the context of modern dynamic stochastic general equilibrium (DSGE) models.

Here we perform such an exploration in the context of a model economy featuring two distinct financial frictions: borrowing constraints and endogenous bank-lending spreads. In this way, our setup nests two of the most prominent hypotheses in the macroeconomic literature on financial frictions: one that emphasizes the role of endogenous collateral constraints (Kiyotaki and Moore, 1997; Iacoviello, 2005), and another one that stresses the role of endogenous lending spreads, as exemplified by the "financial accelerator" literature (Bernanke and Gertler, 1989; Bernanke, Gertler and Gilchrist, 1999) and by the recent literature on banking and macroeconomics (Goodfriend and McCallum, 2007).

Specifically, we consider an economy in which consumers are divided into households and entrepreneurs, where the former are assumed to be relatively more patient and therefore act as savers. The latter do not lend directly to entrepreneurs. Instead, they provide banks with deposits that are then used to make loans to entrepreneurs. Banks are assumed to have some monopolistic power in the loans market. In particular, following Andrés and Arce (2008) we assume that a fixed number of identical banks compete to attract investors as in the spatial competition model of Salop (1979).[1] In this framework, each bank is able to charge a positive lending spread on the deposit rate. Entrepreneurs face endogenous credit limits that link their borrowing capacity to the expected value of their real estate holdings, which we assume is the only collateralizable asset available. Real estate can also be used by entrepreneurs as a production factor in the form of commercial real estate, and by households in the form of residential housing.

In our framework, borrowers optimally select their lending bank and the size of the loan. Both decisions determine the two margins of the demand for loans: the extensive margin (i.e. the market share of each bank) and the intensive margin (i.e. the volume of funds demanded by each borrower). Banks set their profit-maximizing lending rates taking into account that a higher lending rate raises unit margins but reduces both the demand for funds by each borrower and their market share. Due to the effect of collateral constraints, the entrepreneurs' demand for loans depends not only on loan rates but also on the expected growth rate of housing prices and on the pledgeability ratio (which is understood as the maximum fraction of the expected value of a borrower's real estate holdings that can be pledged as collateral). These factors, along with the number of competing banks, are the major determinants of the optimal

[1] Andrés and Arce (2008) analyze the macroeconomic effects of imperfect banking competition and collateral constraints from a positive perspective. Here, we perform a normative analysis within a simplified version of their model.

lending margins, through their influence on the elasticity of the demand schedule faced by each individual bank. In particular, lending margins are lower when housing prices are expected to rise, when borrowing constraints are looser, or when there is a larger pool of banks. Further, there is also a positive relationship between the monetary policy rate, which corresponds to the banks' marginal cost, and the lending margin. We show that our model generates a monetary policy accelerator, since a change in the policy rate produces a more than proportional change in the lending rate. Also, our economy features two familiar nominal frictions: nominal (non-state-contingent) debt and staggered nominal price adjustment à la Calvo (1983), both of which open two additional channels of influence for monetary policy.

Our main focus in this paper is to understand the nature of optimal monetary policy in this framework. To this aim we follow the linear-quadratic approach pioneered by Rotemberg and Woodford (1997) and extensively applied by Woodford (2003) and Benigno and Woodford (2003), among others. This approach consists of deriving a second-order approximation to the aggregate welfare criterion, and a first-order approximation to the equilibrium conditions. The central bank's optimal monetary policy commitment is the one that maximizes the quadratic welfare criterion subject to the linear equilibrium constraints. This approach allows us to clarify what the stabilization goals of the central bank are, and what trade-offs exist between these goals.

Specifically, we show that the central bank's quadratic welfare criterion features four stabilization goals: *inflation*, the *output gap*, the differences in consumption between households and entrepreneurs (*consumption gap*) and the inefficiency in the distribution of real estate between both groups (or *housing gap*). The first two, inflation and the output gap, are related to the existence of staggered price adjustment and are therefore standard in the New Keynesian literature. The last two are novel and are directly related to the existence of financial frictions. First, borrowing constraints prevent constrained consumers from smoothing their consumption the way unconstrained consumers do, thus giving rise to inefficient risk sharing between both consumer types. Second, the distribution of the housing stock between both groups will generally be inefficient, due to the distortionary effect of collateral constraints on entrepreneurs' demand for real estate. In turn, the magnitude of these two sources of inefficiency is amplified by the existence of banks with monopolistic power.

Regarding the linear equilibrium constraints, we show that the consumption gap arises as an endogenous cost-push term in the New Keynesian Phillips curve; that is, inefficient risk-sharing creates a short-run trade-off between inflation and output gap. In addition, we show that closing the consumption and housing gaps requires inefficient fluctuations in inflation and the output gap. Therefore, the optimal policy commitment must yield a compromise between stabilization goals.

Once our model is calibrated, we analyze how the stabilization goals and other variables of interest respond both to productivity shocks and credit-crunch shocks (in the form of exogenous reductions in the pledgeability ratio). We consider the responses under the optimal policy commitment, as well as under simple policy rules such as inflation targeting. Our results can be summarized as follows. Following a negative productivity shock, real estate prices fall and entrepreneurs' consumption (which is tied to their net worth) falls substantially more than the consumption of unconstrained households. To prevent a large consumption gap from arising, the central bank finds it optimal to allow for surprise increases in inflation and the output gap. This way, it deflates the entrepreneurs' real debt burden and increases their profits, hence ameliorating the negative impact of the shock on their net worth and consumption. The narrowing of the consumption gap in turn improves the output-inflation trade-off in subsequent periods. Following a credit-crunch shock, real estate demand by entrepreneurs reacts strongly and a large housing gap arises. In order to close the later, the central bank must again allow for deviations in inflation and output gap; the resulting adjustment in entrepreneurs' net worth allows them to smooth their demand for real estate.

For each policy rule and shock type, we also calculate the volatility of stabilization goals and the associated average welfare loss. For instance, conditional on productivity shocks with a standard deviation of 1%, welfare losses under the optimal commitment amount to 0.03% of steady-state consumption, compared to 0.11% under inflation targeting and 0.09% under output gap targeting. We also analyze the effect of varying the degree of competition in the banking industry on the severity of the policy trade-offs and the resulting welfare losses. Regardless of the nature of the shock, we find that an increase in banking competition increases average welfare losses both under the optimal commitment and (especially) under suboptimal policy rules. Conditional on productivity shocks, welfare losses increase from 0.03% to 0.05% of steady-state consumption under the optimal commitment, and from 0.11% to 0.92% under inflation targeting. The reason is that, as lending spreads fall, entrepreneurs can borrow more for given real estate holdings, thus driving their leverage ratio up. Higher leverage, in turn, implies that fluctuations in real estate prices have stronger effects on entrepreneurs' net worth and hence on the consumption gap. A similar mechanism operates conditional on credit-crunch shocks. However, in this case the increase in welfare losses is proportionately much smaller (from 0.03% to 0.04% under the optimal commitment, and from 0.04% to 0.05% under inflation targeting). The reason is that, under imperfect banking competition, lending margins react counter-cyclically to credit-crunch shocks and thus amplify their effects; under perfect competition, lending margins are zero and their amplifying role disappears.

In a recent paper, Monacelli (2007) simulates the Ramsey optimal monetary policy in a model with collateral constraints and quadratic price adjustment costs. He finds that optimal inflation volatility depends on some preference parameters and the degree of price rigidity. In term of focus and methodology, Cúrdia and Woodford (2008; CW, for short) is the closest reference to this paper. They also focus on the design of optimal monetary rules in the context of a model economy in which a positive spread exists between lending and deposit rates. CW assume that the lending spread is determined by an ad-hoc function of banks' loan volume, aimed at capturing the costs of originating and monitoring loans. We differ from CW in two important respects regarding the nature of credit frictions. First, we model credit spreads as arising endogenously in an environment in which banks enjoy some monopolistic power in the loans market. Second, we subject borrowers to endogenous collateral constraints, thus providing a link between asset prices and the borrowing capacity of constrained consumers. As in CW, we cast our optimal policy problem in a linear-quadratic representation, motivated by its potential for delivering analytical results. Importantly, both CW and our paper find that cyclical fluctuations in lending spreads have minor quantitative effects on the nature of optimal monetary policy design. However, in our model the average level of lending spreads has important quantitative effects that arise from the interaction between spreads and collateral constraints, a channel which is missing in CW. We conclude that the welfare loss incurred when deviating from the optimal policy is sensitive with respect to the level of lending spreads.

The rest of the paper is organized as follows. Section 2 contains the model. In section 3 we analyze the efficient equilibrium which corresponds to the normative benchmark for the optimizing central bank. We derive the optimizing monetary policy criterion in section 4 and discuss the several trade-offs faced by the monetary authority. In section 5 we use some calibrated versions of the model to perform a number of quantitative exercises in order to illustrate the working of optimal and suboptimal monetary rules. Section 6 concludes.

2 Model

In this section we describe a model economy that relies on Iacoviello (2005) and Andrés and Arce (2008). The population of consumers, whose size is normalized to 1, is composed of two types of agents: there is a fraction ω of households and a fraction $1-\omega$ of entrepreneurs. The later are assumed to be more impatient than the former and, in the class of equilibria we analyze later, they borrow up to a limit proportional to the expected present-discounted resale value of their real estate holdings at the time of repaying debts, which is assumed to be one period ahead.

2.1 Households

The representative household maximizes its welfare criterion,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \frac{(l_t^s)^{1+\varphi}}{1+\varphi} + \vartheta_t \log h_t \right),$$

where c_t are units of a Dixit-Stiglitz basket of final consumption goods, l_t^s is labor supply, h_t is residential housing, $\vartheta_t = \vartheta \exp(z_t^h)$ is an exogenously time-varying weight on utility from housing services (where z_t^h follows a zero-mean AR(1) process) and $\beta \in (0,1)$ is the household's subjective discount factor. (Unless otherwise indicated, we use lower case for real variables and upper case for nominal variables. All quantities are expressed in per capita terms.)

Maximization is subject to the following budget constraint expressed in real terms,

$$w_t l_t^s + \gamma_t + s_t + \frac{R_{t-1}^d}{\pi_t} d_{t-1} = c_t + p_t^h \left[\left(1+\tau^h\right) h_t - h_{t-1} \right] + d_t,$$

where w_t is the hourly wage, γ_t are dividends from the corporate sector (other than entrepreneurs), and s_t are lump-sum subsidies from the government. We assume that nominal, risk-free, one-period bank deposits are the only financial asset available to households, where d_t is the real value of deposits at the end of period t, R_t^d is the gross nominal deposit rate and $\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, where P_t is the Dixit-Stiglitz aggregate price index. Households can also buy and sell real estate for residential purposes, where h_t are housing units, p_t^h is the unit price of housing in terms of consumption goods and τ^h is a tax rate on housing purchases by households (the role of which is discussed later on). We assume that real estate does not depreciate. The first order conditions of the problem above are standard:

$$w_t = c_t \left(l_t^s\right)^{\varphi}, \tag{1}$$

$$\frac{1}{c_t} = \beta R_t^d E_t \left\{ \frac{1}{c_{t+1}} \frac{P_t}{P_{t+1}} \right\}, \tag{2}$$

$$\frac{\left(1+\tau^h\right) p_t^h}{c_t} = \frac{\vartheta_t}{h_t} + \beta E_t \frac{p_{t+1}^h}{c_{t+1}}. \tag{3}$$

2.2 Entrepreneurs (intermediate good producers)

Entrepreneurs produce a homogenous intermediate good that is sold under perfect competition to a final goods sector. They operate a Cobb-Douglas production technology,

$$y_t = e^{a_t} \left(l_t^d\right)^{1-\nu} \left(h_{t-1}^e\right)^{\nu}, \tag{4}$$

where y_t is output of the intermediate good, l_t^d is labor demand, h_{t-1}^e is the stock of commercial real estate and a_t is a zero-mean AR(1) exogenous productivity process. Entrepreneurs also demand consumption goods and loans. The budget constraint of the representative entrepreneur is given by

$$b_t + (1-\tau^e)\left(p_t^I y_t - w_t l_t^d\right) = c_t^e + p_t^h (h_t^e - h_{t-1}^e) + \frac{R_{t-1}^e}{\pi_t} b_{t-1}, \tag{5}$$

where b_t is the real value of nominal loans at the end of period t, R_t^e is the gross nominal loan rate, p_t^I is the real price of the intermediate good, τ^e is a tax rate on entrepreneur profits (the role of which is explained below) and c_t^e is entrepreneur consumption. Banks impose a collateral constraint on entrepreneurs: the nominal loan gross of interest payments cannot exceed a certain fraction (the pledgeability ratio) of the expected nominal resale value of the entrepreneur's real estate holdings. The collateral constraint can be expressed in real terms as,

$$b_t \leq m_t E_t \frac{\pi_{t+1}}{R_t^e} p_{t+1}^h h_t^e, \tag{6}$$

where $m_t = m \exp(z_t^m)$ is the exogenously time-varying pledgeability ratio and z_t^m is a zero-mean AR(1) process. In order to obtain a loan, the entrepreneur must first travel to a bank, incurring a utility cost which is proportional to the distance between his and the bank's location. We assume that entrepreneurs and banks are uniformly distributed on a circle of length one. Subject to (4), (5) and (6), an entrepreneur located at point $k \in (0,1]$ maximizes

$$E_0 \sum_{t=0}^{\infty} (\beta^e)^t (\log c_t^e - \alpha d_t^{k,i}),$$

where $d_t^{k,i}$ is the distance between the entrepreneur and the lending bank which is denoted by $i \in \{1, 2, ..., n\}$, and α is the utility cost per distance unit. Entrepreneurs are assumed to be more impatient than savers, *i.e.* $\beta^e < \beta$. The first order conditions of this problem are

$$w_t = p_t^I (1-\nu) \frac{y_t}{l_t^d}, \tag{7}$$

$$\frac{1}{c_t^e} = \beta^e R_t^e E_t \left\{ \frac{1}{c_{t+1}^e} \frac{P_t}{P_{t+1}} \right\} + \xi_t, \tag{8}$$

$$\frac{p_t^h}{c_t^e} = E_t \frac{\beta^e}{c_{t+1}^e} \left\{ (1-\tau^e)\, p_{t+1}^I \nu \frac{y_{t+1}}{h_t^e} + p_{t+1}^h \right\} + \xi_t m_t E_t \frac{\pi_{t+1}}{R_t^e} p_{t+1}^h, \tag{9}$$

where ξ_t is the Lagrange multiplier on the collateral constraint and $p_t^I \nu y_t / h_{t-1}^e$ is the marginal revenue product of commercial real estate. When binding ($\xi_t > 0$), the collateral constraint has two effects on the entrepreneurs decisions: first, it prevents them from smoothing their consumption the way households do (equation (8)); second, it increases the marginal value of real estate due to its role as collateral (equation (9)).

Due to the assumption that $\beta^e < \beta$, it is easy to check that the borrowing constraint binds in the steady state. Provided the fluctuations in the endogenous variables around their steady state are sufficiently small, the borrowing constraint will also bind along the dynamics; that is, equation (6) holds with equality. It is then possible to show that the entrepreneur simply consumes a fraction $1-\beta^e$ of her real net worth,[2]

$$c_t^e = (1-\beta^e) \left[(1-\tau^e)\, \nu p_t^I y_t + p_t^h h_{t-1}^e - \frac{R_{t-1}^e}{\pi_t} b_{t-1} \right], \tag{10}$$

where $(1-\tau^e)\,\nu p_t^I y_t$ are after-tax real profits, $p_t^h h_{t-1}^e$ is commercial real estate wealth and $\left(R_{t-1}^e/\pi_t\right) b_{t-1}$ are real debt repayments.

2.3 Banks

Banks are assumed to intermediate all credit flows between households (savers) and entrepreneurs (borrowers). We assume that banks are perfectly competitive on the deposits market, but competition in the loans market is imperfect so that each bank enjoys some monopolistic power. In order to model imperfect competition in the loans market we use a version of Salop's (1979) circular-city model. Banks are located symmetrically on the unit circle and their position is time-invariant, whereas entrepreneurs' locations vary each period according to an *iid* stochastic process.[3] Bank i chooses the gross nominal interest rate on its loans, $R_t^e(i)$, to maximize

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{c_t}{c_{t+s}} \frac{\Omega_{t+s}(i)}{P_{t+s}}$$

[2] See the proof in the Appendix.

[3] This last assumption removes the possibility that banks exploit strategically the knowledge about the current position of each entrepreneur to charge higher rates in the future.

where $\beta^s c_t/c_{t+s}$ is the time $t+s$ stochastic discount factor of the bank's owners (the households) and $\Omega_{t+s}(i)$ is the bank's nominal profit flow. Denoting by $B_t(i)$ and $D_t(i)$ the nominal stock of loans and deposits of bank i at the end of time t, respectively, we can write its flow of funds constraint as

$$\Omega_t(i) + B_t(i) + R^d_{t-1} D_{t-1}(i) = R^e_{t-1}(i) B_{t-1}(i) + D_t(i).$$

Further, bank i must also obey the balance-sheet identity, $D_t(i) = B_t(i)$. To solve for the bank's optimal loan rate, it is convenient to express its real loan volume as

$$\frac{B_t(i)}{P_T} = b_t(i)\tilde{b}_t(i),$$

where $b_t(i)$ is the *intensive* business margin (the size of each loan) and $\tilde{b}_t(i)$ is the *extensive* business margin (the number of customers, or market share).[4] The first order condition of this problem can be written as

$$R^e_t(i) = R^d_t + \frac{1}{\Lambda_t(i) + \tilde{\Lambda}_t(i)}, \tag{11}$$

where, $\Lambda_t(i) \equiv [-\partial b_t(i)/\partial R^e_t(i)] / b^i_t$ is the semi-elasticity of the intensive business margin and $\tilde{\Lambda}_t(i) \equiv [-\partial \tilde{b}_t(i)/\partial R^e_t(i)]/\tilde{b}_t(i)$ is the semi-elasticity of the extensive business margin. Thus the spread between the lending and the deposit rate is a negative function of the bank's market power, as represented by the semi-elasticities of the intensive margin and the market share.

As shown in Andrés and Arce (2008), in a symmetric equilibrium (i.e. $R^e_{t-1}(i) = R^e_{t-1}\ \forall i$), the optimal lending rate can be expressed as

$$R^e_t = R^d_t + \frac{R^d_t - m_t E_t\left(\pi_{t+1} p^h_{t+1}/p^h_t\right)}{\eta m_t E_t\left(\pi_{t+1} p^h_{t+1}/p^h_t\right) - R^d_t} R^d_t, \tag{12}$$

where

$$\eta \equiv 1 + \frac{n}{\alpha^e}\frac{\beta^e}{1-\beta^e}.$$

Therefore, the lending spread is decreasing in expected house price inflation, $E_t\left(\pi_{t+1} p^h_{t+1}/p^h_t\right)$, the pledgeability ratio, m_t, and the degree of banking competition, as captured by the ratio n/α^e; and it is increasing in the nominal deposit rate, R^d_t. The intuition for these effects is the following. An increase in expected house price inflation or the pledgeability ratio increase entrepreneurs' borrowing capacity. As their indebtedness rises, their demand for loans becomes more elastic, which reduces banks' market power and compresses lending spreads. Similarly,

[4] See Andrés and Arce (2008) for analytical derivations of both margins.

as entrepreneurs become more indebted, the utility cost of servicing the debt becomes more important in the choice of bank relative to the distance utility cost. As a result, small changes in loan rates lead to large flows of customers in search for the lowest loan rate. This pushes lending spreads down. Similar intuitions can be provided regarding the effects of the nominal deposit rate. Finally, an increase in the degree of banking competition compresses lending spreads through an increase in the elasticity of banks' market share with respect to the lending rate.

2.4 Final goods producers

There exist a measure-one continuum of firms that purchase the intermediate good from entrepreneurs and transform it one-for-one into differentiated final goods or varieties. For these firms, the real price of the intermediate good, p_t^I, represents the real marginal cost. Cost minimization by households implies that each final good producer $j \in [0,1]$ faces the following demand curve for its product variety,

$$y_t^f(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} y_t^f, \tag{13}$$

where $P_t(j)$ is the firm's nominal price, $\varepsilon > 1$ is the elasticity of substitution between final good varieties and

$$y_t^f = \omega c_t + (1-\omega)\, c_t^e \tag{14}$$

is the aggregate demand for final goods. As is standard in the New Keynesian literature, we assume staggered nominal price adjustment à la Calvo (1983). Letting θ denote the constant probability of price adjustment, the optimal price decision of the price-setting firms is given by

$$E_t \sum_{T=t}^{\infty} (\beta\theta)^{T-t} \frac{c_t}{c_T} \left\{ (1+\tau)\frac{\tilde{P}_t}{P_T} - \frac{\varepsilon}{\varepsilon-1} p_t^I \right\} P_T^{\varepsilon} y_T^f = 0, \tag{15}$$

where $\tau > 0$ is a subsidy rate on the revenue of final goods producers (the role of which is explained below) and $\tilde{P}_t$ is the optimal price decision. Under Calvo price adjustment, the aggregate price index evolves as follows,

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\, \tilde{P}_t^{1-\varepsilon}\right]^{1/(1-\varepsilon)}. \tag{16}$$

2.5 Market clearing

Total supply of the intermediate good equals $(1-\omega)\, y_t$. Total demand from final good producers equals $\int_0^1 y_t^f(j)dj$, where each firm's demand is given by (13). Equilibrium in the intermediate good market therefore requires

$$(1-\omega)\, y_t = \Delta_t y_t^f, \tag{17}$$

where $\Delta_t \equiv \int_0^1 (P_t(j)/P_t)^{-\varepsilon}\, dj$ is a measure of price dispersion in final goods. Notice that price dispersion increases the amount of the intermediate good that must be produced in order to satisfy a certain level of final consumption demand.

Equilibrium in the housing market implies

$$\bar{h} = \omega h_t + (1-\omega)\, h_t^e, \tag{18}$$

where $\bar{h}$ is the fixed aggregate stock of real estate.

The labor market equilibrium condition is

$$\omega l_t^s = (1-\omega)\, l_t^d. \tag{19}$$

2.6 Monetary policy

The model is closed by means of a monetary policy rule.[5] The latter can be a simple rule, such as strict inflation targeting, or a policy that is optimal with respect to some criterion. Sections 4 and 5 below are devoted to characterizing both types of policy rules and their effects on equilibrium allocations.

3 Efficient equilibrium

In this section we analyze the efficient equilibrium in our model, which will be the normative benchmark for the monetary authority. We assume that, when maximizing aggregate welfare, the social planner assigns to entrepreneurs the same discount factor as that of households, β.[6]

[5] Regarding the budget constraint of the government, by Walras Law such a constraint is implied by all market equilibrium conditions and the flow of funds constraints of all agents in the economy. In particular, the public surplus is the sum of tax receipts from house purchases and entrepreneur profits, minus subsidies to final goods producers. The resulting surplus is rebated to households in the form of lump-sum subsidies, s_t.

[6] Otherwise, the social planner equilibrium would assign less and less consumption to the impatient consumers (the entrepreneurs) relative to the patient ones (the households) as time went by. This would make the

The social planner therefore maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \omega \left[\log(c_t) - \frac{(l_t^s)^{1+\varphi}}{1+\varphi} + \vartheta_t \log(h_t) \right] + (1-\omega) \log(c_t^e) \right\}$$

subject to the aggregate resource constraints for consumption goods and real estate,

$$(1-\omega) e^{a_t} \left(h_{t-1}^e\right)^{\nu} \left(\frac{\omega}{1-\omega} l_t^s \right)^{1-\nu} = \omega c_t + (1-\omega) c_t^e, \tag{20}$$

$$\bar{h} = \omega h_t + (1-\omega) h_t^e, \tag{21}$$

where we have used equation (19) to substitute for l_t^d in the left-hand side of equation (20). Using equations (20) and (21) to solve for c_t and h_t, respectively, the social-planner problem simplifies to the choice of the optimal state-contingent path of c_t^e, h_t^e and l_t^s. The first-order conditions of this problem can be expressed as

$$c_t = c_t^e, \tag{22}$$

$$\beta E_t \frac{1}{c_{t+1}} \nu \frac{e^{a_{t+1}} \left(h_t^e\right)^{\nu} \left[\omega l_{t+1}^s / (1-\omega)\right]^{1-\nu}}{h_t^e} = \frac{\vartheta_t}{h_t}, \tag{23}$$

$$c_t \left(l_t^s\right)^{\varphi} = \frac{1-\omega}{\omega} (1-\nu) \frac{y_t}{l_t^s}. \tag{24}$$

Notice that equations (20) and (22) jointly imply $(1-\omega) e^{a_t} \left(h_{t-1}^e\right)^{\nu} \left[\omega l_t^s / (1-\omega)\right]^{1-\nu} = c_t$. Using this in equation (23), we have that the efficient distribution of the housing stock is given by

$$\frac{h_t^e}{h_t} = \frac{\beta \nu}{(1-\omega) \vartheta_t}. \tag{25}$$

This, combined with equation (21), implies the following solution for aggregate residential housing,

$$\omega h_t = \frac{\omega \vartheta_t}{\omega \vartheta_t + \beta \nu} \bar{h}. \tag{26}$$

Using equations (20) and (22) in equation (24) we obtain the following solution for efficient labor supply,

$$l_t^s = \left(\frac{1-\nu}{\omega} \right)^{1/(1+\varphi)}. \tag{27}$$

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equilibrium allocation dependent on the time elapsed since the implementation of the social planner solution, and the system of equations would not have a recursive representation.

Substituting this expression into the production function, we obtain the efficient level of output,

$$y_t = e^{a_t} \left(h_{t-1}^e \right)^{\nu} \left(\frac{\omega}{1-\omega} \right)^{1-\nu} \left[\frac{1-\nu}{\omega} \right]^{(1-\nu)/(1+\varphi)} . \tag{28}$$

To summarize, the efficient equilibrium is characterized by full consumption risk sharing between households and entrepreneurs (equation (22)), a distribution of the housing stock that changes only with shocks to housing utility (equation (25)) and a constant labor supply (equation (27)).[7] These features will help us understand the stabilization goals and trade-offs of monetary policy. We turn to this now.

4 Optimal monetary policy

In order to analyze optimal monetary policy, we follow the *linear-quadratic* approach pioneered by Rotemberg and Woodford (1997) and extensively applied in Woodford (2003). The linear-quadratic method consists of deriving a log-quadratic approximation of aggregate welfare (which will represent the objective function of the central bank) and a log-linear approximation of the equilibrium conditions (which will be the constraints on the central bank's optimization problem). As is well known, this method is helpful at clarifying what the stabilization goals are for the central bank and what trade-offs exist among those goals. Indeed, the application of this method in our setup delivers a set of analytical results that facilitate greatly the interpretation of our subsequent numerical results.

4.1 Quadratic loss function

As emphasized by Benigno and Woodford (2008), the approximation of the aggregate welfare criterion must be purely quadratic in order for the linear-quadratic approach to provide a correct welfare ranking (with an accuracy of up to second order) of alternative monetary policy rules. Derivation of a purely quadratic approximation is greatly simplified by the assumption of an efficient steady state for the welfare-relevant variables. As shown in the Appendix, steady-state efficiency for such variables can be implemented in our framework by making the following three assumptions.

[7] The fact that neither labor hours nor the distribution of real estate are affected by productivity shocks in the efficient equilibrium is due to our assumption of logarithmic utility of consumption. Deviating from the latter assumption would complicate the algebra without adding much to our main insights about the nature of optimal monetary policy.

Assumption 1 *The subsidy rate on the revenue of final goods producers is given by*

$$\tau = \frac{\varepsilon}{\varepsilon - 1} - 1 > 0.$$

Assumption 2 *The tax rate on entrepreneur profits is given by*

$$\tau^e = 1 - \frac{1-\omega}{(1-\beta^e)\,\nu} \frac{1 - \beta^e - m\,(1/R^e_{ss} - \beta^e)}{1 - m/R^e_{ss}}.$$

Assumption 3 *The subsidy rate on residential housing purchases is given by*

$$\tau^h = \frac{\beta}{\beta^e} \frac{1 - \beta^e - m\,(1/R^e_{ss} - \beta^e)}{1 - \tau^e} - 1 + \beta.$$

The first assumption eliminates the monopolistic distortion in final goods markets, such that steady-state real marginal costs are unity ($p^I_{ss} = 1$). The second assumption guarantees efficient risk-sharing between households and entrepreneurs in the steady state ($c_{ss} = c^e_{ss}$). The third one implements the efficient steady-state distribution of real estate between commercial and residential uses ($h^e_{ss}/h_{ss} = \beta\nu/\left[(1-\omega)\,\vartheta\right]$). Under assumptions 1 to 3, aggregate welfare can be approximated by[8]

$$\sum_{t=0}^{\infty} \beta^t \left\{ \omega \left[\log c_t + \vartheta_t \log h_t - \frac{(l^s_t)^{1+\varphi}}{1+\varphi} \right] + (1-\omega) \log c^e_t \right\} = -\sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O^3,$$

where $t.i.p.$ are terms independent of policy, O^3 are terms of order third and higher, and

$$L_t = \lambda_\pi \hat{\pi}_t^2 + \lambda_y \left(\hat{y}_t - \hat{y}_t^{*|h} \right)^2 + \lambda_c \left(\hat{c}_t - \hat{c}^e_t \right)^2 + \lambda_h \left(\hat{h}_t - \hat{h}^*_t \right)^2 \tag{29}$$

is a purely quadratic period loss function, where hats denote log-deviations from steady state and weight coefficients are given by

$$\lambda_\pi \equiv \frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)}, \lambda_y \equiv \frac{1+\varphi}{1-\nu}, \lambda_c \equiv \omega\,(1-\omega)\,, \lambda_h \equiv \omega\vartheta \frac{\omega\vartheta + \beta\nu}{\beta\nu}.$$

The loss function illustrates the existence of four stabilization goals for the central bank. The first one is inflation. As is well known, under staggered price adjustment inflation creates

[8] See the proof in the Appendix.

inefficient price dispersion and hence a welfare loss. The second goal is the output gap, where

$$\hat{y}_t^{*|h} \equiv a_t + \nu \hat{h}_{t-1}^e$$

is efficient output conditional on the stock of commercial housing (see equation (28)).[9] Nominal price rigidities produce inefficient fluctuations in output which, given the stock of commercial property, generates in turn inefficient fluctuations in labor hours. These first two goals are standard in the New Keynesian model.

The third and fourth goals are directly related to the existence of credit frictions in this model. The third goal is the (log) difference between the consumption of households and entrepreneurs, i.e. between unconstrained and constrained consumers, which we may refer to as the *consumption gap*. This term captures the aggregate welfare losses produced by inefficient risk sharing between households and entrepreneurs, which is in turn the result of collateral constraints on entrepreneurs. The fourth goal is the (log) difference between the actual and the efficient level of residential real estate, or *housing gap*, where

$$\hat{h}_t^* \equiv \frac{\beta\nu}{\omega\vartheta + \beta\nu} z_t^h \tag{30}$$

is efficient residential housing (see equation (26)). Notice that, given the fixed supply of real estate, an inefficient level of residential real estate is equivalent to an inefficient distribution of real estate between residential and commercial uses. Inefficiency in the real estate distribution in our framework can arise for various reasons. First, demand of commercial property by entrepreneurs is distorted by its role as collateral in loan agreements. Second, since households and entrepreneurs differ in their degree of impatience and their consumption, they also use different stochastic discount factors for pricing future state-contingent payoffs from housing. Finally, the fact that commercial real estate purchases are partly financed by loans implies that they are prone to suffering the effects of credit-crunch shocks.

4.2 Policy trade-offs

The second step of the linear-quadratic approach consists of log-linearizing the equilibrium conditions around the steady state. For brevity, the complete list of log-linear equations is

[9] This is different from *un*conditionally efficient output, which depends on the path of commercial housing that would have been observed in an efficient equilibrium. For a discussion on this issue, see Woodford (2003, section 5.3) and Neiss and Nelson (2003).

deferred to the Appendix.[10] Here, we restrict our attention to those equations that are helpful for understanding the trade-offs among stabilization goals. We start by log-linearizing and combining equations (15) and (16), which yields

$$\hat{\pi}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{p}_t^I + \beta E_t \hat{\pi}_{t+1}. \tag{31}$$

In order to find an expression for real marginal costs, $\hat{p}_t^I$, we first log-linearize equations (1), (7) and (19), and combine them into

$$\hat{c}_t + \varphi \hat{l}_t^s = \hat{p}_t^I + \hat{y}_t - \hat{l}_t^s. \tag{32}$$

That is, the labor supply schedule (the marginal rate of substitution between consumption and leisure) must intersect the labor demand schedule (the marginal revenue product of labor). Second, we log-linearize the production function and solve for labor hours, obtaining

$$\hat{l}_t^s = \frac{1}{1-\nu}\left(\hat{y}_t - \hat{a}_t - \nu \hat{h}_{t-1}^e\right) = \frac{1}{1-\nu}\left(\hat{y}_t - \hat{y}_t^{*|h}\right). \tag{33}$$

Third, we log-linearize the equilibrium conditions in the final goods and intermediate good markets, equations (14) and (17) respectively, and combine them into

$$\hat{y}_t = \omega \hat{c}_t + (1-\omega)\hat{c}_t^e, \tag{34}$$

where we have used the fact that $(1-\omega) y_{ss} = c_{ss} = c_{ss}^e$ in the efficient steady state.[11] Combining equations (32) to (34), we can express real marginal costs as

$$\hat{p}_t^I = \frac{1+\varphi}{1-\nu}\left(\hat{y}_t - \hat{y}_t^{*|h}\right) + (1-\omega)(\hat{c}_t - \hat{c}_t^e). \tag{35}$$

Using this in equation (31) yields the following *New Keynesian Phillips curve*,

$$\hat{\pi}_t = \kappa \frac{1+\varphi}{1-\nu}\left(\hat{y}_t - \hat{y}_t^{*|h}\right) + \beta E_t \hat{\pi}_{t+1} + \kappa (1-\omega)(\hat{c}_t - \hat{c}_t^e), \tag{36}$$

[10] Simulation results not reported here indicate that the Ramsey optimal long-run gross rate of inflation is $\pi_{ss} = 1$, regardless of whether the steady state is assumed to be efficient or not. Therefore, our log-linearization is performed around a zero net inflation steady state. The reason for this result is essentially the same as the reason why the optimal long-run net rate of inflation is zero in the standard New Keynesian model, namely that the welfare losses of committing to positive inflation rates in the future outweigh the welfare gains of exploiting the short-run output-inflation trade-off when output is inefficiently low (see e.g. Woodford, 2003).

[11] As shown in the Appendix, the price dispersion term Δ_t is of second order and therefore drops out of the log-linear approximation of equation (17).

where $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$. Equation (36) has the same form as the standard New Keynesian Phillips curve, with the exception of the last term on the right hand side, which is proportional to the consumption gap. Therefore, collateral constraints and the resulting inefficient risk-sharing create an endogenous trade-off between output gap and inflation. The reason is the following. From equation (32), real marginal costs $\hat{p}_t^I$ depend on labor hours and the difference between aggregate demand and household consumption. Because of inefficient risk sharing, fluctuations in aggregate demand and household consumption will be unequal. As a result, keeping labor hours constant (that is, closing the output gap) is not enough to prevent fluctuations in real marginal costs and hence in inflation.

From the preceding analysis, it follows that closing the consumption gap has two beneficial effects on aggregate welfare. First, it improves the trade-off between inflation and output gap. Second, since the consumption gap is itself a stabilization goal, closing it has a direct beneficial effect on welfare. A third normative reason for closing the consumption gap is that it relaxes the borrowing constraint on entrepreneurs. This makes the real estate distribution more efficient over the cycle and thus helps closing the housing gap.

While desirable, consumption gap stabilization requires itself inefficient fluctuations in other stabilization goals. To see this, consider the log-linear approximation of the entrepreneur consumption equation around the efficient steady state ($c_{ss}^e/y_{ss} = 1-\omega$),

$$\hat{c}_t^e = (1-\beta^e)\left[\frac{(1-\tau^e)\,\nu}{1-\omega}\left(\hat{p}_t^I + \hat{y}_t\right) + \frac{p_{ss}^h h_{ss}^e}{c_{ss}^e}\left(\hat{p}_t^h + \hat{h}_{t-1}^e\right) - m\frac{p_{ss}^h h_{ss}^e}{c_{ss}^e}\left(\hat{R}_{t-1}^e + \hat{b}_{t-1} - \hat{\pi}_t\right)\right], \tag{37}$$

where both sides have been normalized by c_{ss}^e and we have used $b_{ss}R_{ss}^e = mp_{ss}^h h_{ss}^e$. The borrowing constraint, equation (6) holding with equality, can be approximated by $\hat{R}_t^e + \hat{b}_t = z_t^m + E_t\hat{p}_{t+1}^h + \hat{h}_t^e + E_t\hat{\pi}_{t+1}$. Substituting this into equation (37) and rearranging terms, we obtain

$$\hat{c}_t^e = (1\text{-}\beta^e)\left\{\frac{(1\text{-}\tau^e)\,\nu}{1-\omega}\left(\hat{p}_t^I + \hat{y}_t\right) + \frac{p_{ss}^h h_{ss}^e}{c_{ss}^e}\left[\left(\hat{p}_t^h - mE_{t\text{-}1}\hat{p}_t^h\right) + (1-m)\,\hat{h}_{t\text{-}1}^e + m\left(\hat{\pi}_t - E_{t\text{-}1}\hat{\pi}_t\right) - mz_{t\text{-}1}^m\right]\right\}. \tag{38}$$

Therefore, entrepreneur profits $(\hat{p}_t^I + \hat{y}_t)$, inflation surprises $(\hat{\pi}_t - E_{t-1}\hat{\pi}_t)$ and quasi-surprises in real estate prices $(\hat{p}_t^h - mE_{t-1}\hat{p}_t^h)$ are the major endogenous determinants of entrepreneur consumption. The latter will therefore differ from household consumption, which is driven exclusively by intertemporal substitution considerations. In response to unexpected shocks, it is however possible for the central bank to bring entrepreneur and household consumption closer to each other. First, by engineering inflation surprises it can manipulate the real value of debt repayments and hence the equilibrium path of entrepreneurial net worth. Second, notice that entrepreneur profits can be expressed in terms of stabilization goals as follows,

$$\hat{p}_t^I + \hat{y}_t = \left(\frac{1+\varphi}{1-\nu} + 1\right)\left(\hat{y}_t - \hat{y}_t^{*|h}\right) + (1-\omega)\left(\hat{c}_t - \hat{c}_t^e\right) + \hat{y}_t^{*|h},$$

where we have used equation (35) to substitute for $\hat{p}_t^I$. Therefore, the central bank can also resort to output-gap management so as to manipulate entrepreneur profits in a way that reduces the consumption gap.

To summarize, optimal monetary policy will involve a trade-off between all four stabilization goals. We now turn to the quantitative analysis of these trade-offs.

5 Quantitative analysis

5.1 Calibration

We calibrate our model to quarterly US data. The calibration is largely based on Andrés and Arce (2008) and Iacoviello (2005). The household discount factor, $\beta = 0.993$, is chosen such that the annual real interest rate equals 3%. The entrepreneur discount factor is set to 0.95, within the range of values for constrained consumers typically used in the literature. The elasticity of production with respect to commercial housing, $\nu = 0.05$, is set in order to generate a steady-state ratio of commercial housing wealth to annual output of 62%. Similarly, the weight on housing utility, $\vartheta = 0.11$, is chosen to match an average ratio of residential housing wealth to annual output of 140%. Regarding the banking parameters, what matters for the steady-state level of lending spreads is the ratio n/α. We arbitrarily set the number of banks to 10, and then set the distance utility parameter, α, to obtain a steady-state annual lending spread of 2.5%. The size of the household population, $\omega = 0.979$, is chosen such that the tax rate on entrepreneur profits equals zero in the steady state.[12] The loan-to-value ratio is set to $m = 0.85$, as in Iacoviello (2005). The labor supply elasticity is set to one half, which is broadly consistent with micro evidence. The elasticity of demand curves is set to 6, which would imply a monopolistic mark-up of 20% in the absence of subsidies. The Calvo parameter implies a mean duration of price contracts of 3 quarters, consistent with recent micro evidence (Bils and Klenow, 2004, Nakamura and Steinsson, 2008). The subsidy rates on residential

[12] Alternatively, we could have calibrated ω empirically and then chosen the tax rate τ^e that implements efficient risk-sharing in the steady state. Since the share of entrepreneurs in the population is small, this alternative approach would produce a very similar calibration.

housing purchases that implements the efficient steady state is 1.2%. Finally, the structural parameters imply weights (normalized by their sum) of $\lambda_\pi = 91.2\%$, $\lambda_y = 7.9\%$, $\lambda_c = 0.1\%$ and $\lambda_h = 0.8\%$ in the loss function.

Table 1. Calibration

	Value	Target	Description
β	0.993	$R^d_{ss}/\pi_{ss} = (1.03)^{1/4}$	household discount factor
β^e	0.95	standard	entrepreneur discount factor
ν	0.05	$p^h_{ss}h^e_{ss}/(4y_{ss}) = 0.62$	elasticity of output wrt housing
ϑ	0.11	$p^h_{ss}h_{ss}/(4y_{ss}) = 1.40$	relative weight on housing utility
n, α	10, 6.32	$4(R^e_{ss} - R^d_{ss}) = 2.5\%$	number of banks, distance cost
ω	0.979	$\tau^e = 0$	household share of population
m	0.85	standard	loan-to-value ratio
φ	2	$1/\varphi = 0.5$	(inverse of) labor supply elasticity
ε	6	$(1+\tau)/p^I_{ss} = 1.20$	intratemporal elasticity of subst.
θ	0.67	$1/(1-\theta) = 3$ qrts.	Calvo parameter

5.2 Impulse response analysis

In order to investigate the nature of optimal monetary policy in this framework, we now analyze the economy's response to shocks under the optimal commitment. We consider both productivity shocks as well as *credit-crunch* shocks, in the form of shocks to the pledgeability ratio.[13] We also analyze the impulse-responses under a policy of strict inflation targeting ($\hat{\pi}_t = 0$). Such a policy has been shown to be optimal in the standard New Keynesian model (see e.g. Goodfriend and King, 2001, and Woodford, 2003). By comparing both policies, we can illustrate the trade-offs that render inflation targeting suboptimal in this framework.

5.2.1 Productivity shocks

Figure 1 plots the economy's response to a 1% negative productivity shock, both under the optimal commitment and strict inflation targeting.[14]

FIGURE 1 HERE

[13] For brevity, we omit the results regarding the effects of shocks to the utility of housing services (ϑ_t). These results however are available upon request from the authors.

[14] We assume an autocorrelation coefficient of 0.95.

Let us focus first on the case of strict inflation targeting (dotted lines). The fall in total factor productivity reduces the marginal product of housing, which leads to a fall in entrepreneur profits, their demand for real estate and real estate prices. Lower profits and real estate wealth, in turn, trigger a large fall in entrepreneur net worth and hence in entrepreneur consumption. Household consumption also falls, but it does so by a relatively small amount, thanks to households' ability to smooth consumption. As a result, the consumption gap increases sharply on impact. In addition to lowering welfare, the increase in the consumption gap also shifts the New Keynesian Phillips curve upwards. In order to keep inflation at zero, the central bank is obliged to engineer a drop in the output gap. On the other hand, the fall in entrepreneurs' holdings of real estate produces an increase in the housing gap (remember that productivity shocks do not affect the efficient real estate distribution; see equation (30)). To summarize, strict inflation targeting requires inefficient fluctuations in the output gap and, especially, in the consumption and housing gaps.

Relative to the situation under inflation targeting, the optimal policy can improve matters by allowing for a certain amount of surprise inflation. This way, it can reduce the real value of entrepreneurs' debt burden. At the same time, the central bank engineers an increase in the output gap (that is, it reduces the drop in aggregate demand) also on impact. Both actions have a beneficial effect on entrepreneurs' real net worth and therefore on their consumption. As a result, the consumption gap experiences a substantially smaller increase. This in turn contains the upward shift in the Phillips curve, thus improving the trade-off between inflation and output gap. Indeed, both variables return to zero very quickly.

The optimal policy is implemented via a sharper cut in the nominal interest rate (lower right panel in Figure 1). Such an aggressive monetary response implies that housing prices fall by less than under inflation targeting, both on impact and along the whole recovery path. This way, the optimal policy reduces the cyclical response of credit, which is positively linked to the value of collateral, and hence facilitates a smoother adjustment of entrepreneur consumption. Finally, the endogenous response in lending spreads (not shown in the figure) is very small under both policy regimes, with peak drops of 1 and 0.2 basis points, respectively. This suggests that, given a level of banking competition, the endogenous fluctuations in lending margins do not seem to amplify the effects of productivity shocks.[15]

[15] Andrés and Arce (2008) provide a detailed analysis of the various opposite-sign effects that explain such a low responsiveness of spreads following a productivity shock. Section 5.3 below analyzes the effects of changes in the average (steady-state) levels of bank lending margins.

5.2.2 Credit crunch shocks

Figure 2 plots the impulse-responses to a 1% negative shock to the pledgeability ratio (m_t) under the optimal policy and inflation targeting.[16] Again, we focus first on the case of inflation targeting. The fall in the pledgeability ratio reduces the amount of borrowing by entrepreneurs, and hence their real estate holdings are immediately reduced. This produces a symmetric increase in the housing gap (the efficient real estate distribution is invariant to this shock too; see again equation (30)). Regarding the other stabilization goals, the responses are of minor importance. First, notice that the consumption gap first reacts negatively and then positively.[17] Since the reaction of household consumption is relatively small, the consumption gap basically mirrors the behavior of entrepreneur consumption. And because what matters for inflation dynamics is the entire present-discounted path of consumption gaps, the shift in the Phillips curve is very small, such that a tiny correction in the output gap is enough to keep inflation at zero.

FIGURE 2 HERE

Therefore, the optimal policy is primarily aimed at reducing the housing gap. In order to achieve this, the monetary authority engineers surprise increases in inflation and the output gap. Following the same logic as in the analysis of productivity shocks, both actions have positive effects on entrepreneur net worth. Real estate demand by entrepreneurs is the sum of unmortgaged real estate wealth (which equals β^e times real net worth) plus borrowing. Thanks to the increase in their net worth, entrepreneurs can increase their holdings of unmortgaged real estate. This partially compensates the drop in entrepreneurial borrowing, hence producing a smaller reaction in commercial real estate and the housing gap.

The optimal policy is implemented again by means of a stronger cut in nominal interest rates than under inflation targeting. Regarding lending margins (lower left panel in Figure 2), they experience a non-negligible impact increase of 15 basis points in annualized terms. That is, the response of spreads now is much more pronounced than in the case of a productivity shock since in this case banks take advantage of the fact that credit scarcity goes hand in hand with a more inelastic demand for funds. However, such a response is nearly the same under both scenarios, which indicates that it is mostly due to the direct effect of the shock to m_t (see equation (12)).

[16] We assume an autocorrelation coefficient of 0.75, such that the shock has a half-life of four quarters.

[17] The reason is the following. The fall in the pledgeability ratio has two opposing effects on entrepreneurs' net worth and consumption. First, it reduces their real estate wealth. Second, by reducing entrepreneurs' borrowing it also reduces debt repayments from the second period onwards. As can be seen in figure 2, the first effect dominates on impact but the second one becomes dominant from the third period onwards.

5.3 Volatility of stabilization goals and welfare loss

The previous section characterized the responses of the stabilization goals to productivity and credit-crunch shocks. These goals however enter with different weights in the loss function of the central bank, and therefore have different quantitative effects on welfare. We are ultimately concerned with the welfare implications of alternative monetary policy rules. The next section quantifies the welfare losses that arise under different monetary policy regimes.

5.3.1 Welfare losses under the baseline calibration

The first four columns of Table 2 display the standard deviation of the four stabilization goals, conditional on productivity shocks (with a standard deviation of 1%). As in the previous section, we consider the cases of inflation targeting and the optimal policy commitment. We also include output gap targeting, which is equivalent to inflation targeting in the standard New Keynesian model (that is, the model without an output-inflation trade-off). The last column displays the implied average welfare loss, as a percent of steady-state consumption.

Table 2

Standard deviation of stabilization goals and welfare loss. Productivity shock

Policy rule	$4\hat{\pi}_t$	$\hat{y}_t - \hat{y}_t^*$	$\hat{c}_t - \hat{c}_t^e$	$\hat{h}_t - \hat{h}_t^*$	Welfare loss
inflation targeting	0	0.08	11.67	4.73	0.11
output gap targeting	0.85	0	9.79	3.97	0.09
optimal policy	0.81	0.49	3.49	1.41	0.03

Note: sd in %, welfare loss as a % of steady-state consumption

As the table makes clear, a policy of strict inflation targeting implies large fluctuations in the consumption and housing gaps. Fluctuations in the output gap are rather small. These volatilities, together with the weights in the loss function, imply an average welfare loss of 0.11% of steady-state consumption. Regarding the case of output gap targeting, fluctuations in the consumption and housing gaps are of similar magnitude, whereas (annualized) inflation has a standard deviation of 85 basis points. The implied average welfare loss (0.09%) is close to the one under inflation targeting. Finally, the optimal monetary policy balances all the trade-offs among goals, producing a welfare loss of just 0.03% of steady-state consumption.

Table 3 shows the standard deviation of the stabilization goals and the implied average welfare losses, conditional on shocks to the pledgeability ratio (with a standard deviation of 1%). Again, the larger fluctuations take place in the consumption and housing gaps. Under inflation and output-gap targeting, housing gaps are more volatile than consumption gaps. As we saw in the analysis of impulse-responses, relative to inflation targeting the optimal policy focuses on reducing fluctuations in the housing gap, at the cost of larger fluctuations in all other goals; this is reflected in the standard deviations in Table 3. Finally, in terms of average welfare losses the two suboptimal policy rules are now comparatively closer to the optimal policy than in the case of productivity shocks. Intuitively, credit crunch shocks that raise collateral requirements directly affect the borrowing ability of entrepreneurs and hence their demand for real estate; as a result, the policy of sustaining entrepreneurs' net worth so as to prevent large drops in their real estate holdings becomes less effective. In fact, as illustrated in Table 3, such a policy comes now at the cost of increased volatility of entrepreneur consumption and the consumption gap.

Table 3

Standard deviation of stabilization goals and welfare loss. Credit-crunch shock

Policy rule	$4\hat{\pi}_t$	$\hat{y}_t - \hat{y}_t^*$	$\hat{c}_t - \hat{c}_t^e$	$\hat{h}_t - \hat{h}_t^*$	Welfare loss
inflation targeting	0	0.01	2.19	3.37	0.042
output gap targeting	0.31	0	2.08	3.45	0.046
optimal policy	0.36	0.23	4.94	2.21	0.027

Note: sd in %, welfare loss as a % of steady-state consumption

5.3.2 The effects of banking competition

The welfare analysis can also shed some light on the importance of the intensity of competition of the banking sector. Andrés and Arce (2008) find that variations in the level of banking competition have a moderate effect on the short term impulse response dynamics of the main variables, but a sizeable one in the medium term, due to the interaction of banking competition with the persistence of the net worth.

To isolate the effects of banking competition on monetary policy trade-offs, we repeat our exercises under the assumption of perfect banking competition ($\alpha = 0$, or $n \rightarrow \infty$). This implies that the steady-state loan rate R_{ss}^e falls from its baseline value to $R_{ss}^d = 1/\beta$, and the interest rate spread goes to zero. The effect of this structural change on the fluctuations of the stabilization goals and the associated welfare losses, conditional on productivity shocks, is summarized in Table 4.

Table 4

Banking competition and the sd of stabilization goals and welfare loss.

Productivity shock

Banking regime	$4\hat{\pi}_t$	$\hat{y}_t - \hat{y}_t^*$	$\hat{c}_t - \hat{c}_t^e$	$\hat{h}_t - \hat{h}_t^*$	Welfare loss
		Inflation targeting			
baseline calibration	0	0.08	11.67	4.73	0.11
perfect competition	0	0.27	40.94	12.53	0.92
		Output gap targeting			
baseline calibration	0.85	0	9.79	3.97	0.09
perfect competition	5.36	0	21.94	6.60	0.89
		Optimal policy			
baseline calibration	0.81	0.49	3.49	1.41	0.03
perfect competition	1.12	0.69	3.00	0.64	0.05

Note: sd in %, welfare loss as a % of steady-state consumption

As the table makes clear, the policy trade-offs worsen when the banking industry is more competitive. For the two suboptimal policies, all stabilization goals become more volatile. As a result, average welfare losses increase. In fact, welfare losses now are of first-order importance (0.92% under inflation targeting, 0.89% under output gap targeting). Regarding the optimal policy, the increase in inflation and output-gap volatility is partially offset by smaller fluctuations in the consumption and housing gaps. However, the larger weights of the former two goals in the loss function (91.2% and 7.9%, respectively) implies an increase in average welfare loss. This increase is however small. We conclude that, conditional on productivity shocks, the transit to perfect banking competition makes the policy trade-offs more severe, but especially so for the suboptimal policy rules.

To understand these results, it is helpful to consider the steady-state effects of an increase in banking competition. We start by defining the *leverage ratio* as the fraction of borrowers' asset holdings that are financed with debt, $b_t/\left(p_t^h h_t^e\right)$. The collateral constraint implies that, in the steady state, the leverage ratio equals m/R_{ss}^e. Therefore, the fall in loan rates produced by stronger banking competition increases the leverage ratio. Now consider equation (38), which determines the cyclical fluctuations in entrepreneur (log)consumption. In the latter equation, the term capturing the sensitivity of entrepreneur consumption to real estate prices can be expressed as

$$\frac{p_{ss}^h h_{ss}^e}{c_{ss}^e} = \frac{\beta}{1-\beta} \frac{1}{1 - b_{ss}/\left(p_{ss}^h h_{ss}^e\right)},$$

where we have used the fact that entrepreneurs devote a fraction $1-\beta$ of their real net worth to consumption, c^e_{ss}, and the remaining fraction β to financing the part of real estate holdings that exceeds the amount of borrowing, $p^h_{ss}h^e_{ss} - b_{ss}$. Therefore, the increase in the leverage ratio amplifies the effect of fluctuations in real estate prices on entrepreneur consumption. Since household consumption is not affected by collateral constraints, the increased volatility of entrepreneur consumption carries over to the consumption gap. As we have seen, this has a direct negative effect on welfare, but it also worsens the output-inflation trade-off (by causing larger shifts in the Phillips curve) and amplifies the distortions in the distribution of real estate. Taking all these effects together, we have that the increase in banking competition tends to exacerbate the trade-offs of monetary policy and the associated welfare losses.

Table 5 shows the effects on goal volatility and welfare of increasing banking competition, conditional on credit-crunch shocks. The main message from the table is that stabilization goals also tend to become more volatile, but only by relatively small amounts. As a result, the average welfare loss increases under all the policy rules considered, but the increases are proportionately more modest than in the case of productivity shocks. The intuition for this can be found in the behavior of the lending spread. The latter is zero under perfect banking competition, but in the baseline case it is positive and countercyclical in response to credit-crunch shocks. In the perfect competition case, the amplification mechanism emphasized for productivity shocks applies as well to credit-crunch shocks: the increase in financial leveraging reinforces the effects of housing price fluctuations on consumption and housing gaps.[18] However, under the baseline calibration the lending spread reacts countercyclically to changes in the pledgeability ratio, thus amplifying the negative effect of the shock. These two opposite effects (leverage and interest rate spread) dampen the differences in welfare loss between both scenarios of banking competition.

[18] In addition, according to equation (38) the increase in $p^h_{ss}h^e_{ss}/c^e_{ss}$ also amplifies the lagged effect of credit-crunch shocks, z^m_{t-1}, on entrepreneur consumption.

Table 5

Banking competition and the sd of stabilization goals and welfare loss.

Credit crunch shock

Banking regime	$4\hat{\pi}_t$	$\hat{y}_t - \hat{y}_t^*$	$\hat{c}_t - \hat{c}_t^e$	$\hat{h}_t - \hat{h}_t^*$	Welfare loss
Inflation targeting					
baseline calibration	0	0.01	2.19	3.37	0.042
perfect competition	0	0.02	2.39	3.65	0.049
Output gap targeting					
baseline calibration	0.31	0	2.08	3.45	0.046
perfect competition	0.52	0	1.93	3.80	0.059
Optimal policy					
baseline calibration	0.36	0.23	4.94	2.21	0.027
perfect competition	0.11	0.08	5.66	3.11	0.042

Note: sd in %, welfare loss as a % of steady-state consumption

6 Conclusions

In this paper we provide a theoretical framework for analysis of the optimal conduct of monetary policy in the presence of financial frictions in the form of collateral constraints and a monopolistically competitive banking sector. In our economy consumers are divided into households and entrepreneurs with different time preferences. There is a banking technology that intermediates between savers and borrowers. Banks are assumed to have some monopolistic power in the loans market and set optimal lending rates accordingly. There is only one collateralizable asset that also yields utility and productive returns: real estate. Entrepreneurs face endogenous credit limits that link their borrowing capacity to the expected value of their real estate holdings.

We show that, under the assumption of an efficient steady-state and up to a second order approximation, welfare maximization is equivalent to stabilization of four goals: inflation, output gap, as well as the consumption gap and the distribution of the collateralizable asset between constrained and unconstrained consumers. Following both productivity and credit-crunch shocks (the latter in the form of transitory changes in the pledgeability ratio), the optimal monetary policy commitment implies a short-run trade-off between these goals. Relative to strict inflation targeting, optimal monetary policy requires surprise fluctuations in inflation and output gap, as well as larger adjustments in the nominal policy rate, in order to prevent large fluctuations in entrepreneurs' net worth and hence in the consumption and housing gaps.

The welfare gain of pursuing optimal polices is comparatively higher under technology-driven fluctuations. Shocks to the pledgeability ratio have a direct effect on the borrowing ability of entrepreneurs, such that policies aimed at sustaining their net worth are less effective in preventing large swings in entrepreneurial real estate holdings.

We also compare the nature of these trade-offs under alternative assumptions about the degree of competition in the banking industry. We find that, both under optimal and suboptimal policies, welfare losses due to cyclical fluctuations are amplified as banking competition increases. This amplification is moderate in financially driven (credit-crunch) fluctuations but is substantial if productivity shocks dominate. Key to these results is the interplay between two endogenous mechanisms at work in our model: leverage and lending margins. As banking competition increases, the constrained consumers (entrepreneurs) become more leveraged, thus amplifying the response of their consumption and real estate holdings to exogenous shocks; this worsens the aforementioned trade-offs and makes the use of optimal polices more compelling. The countercyclical response of lending margins to shocks aggravates the policy trade-offs and is stronger in less competitive environments. The first mechanism is equally important regardless of the nature of the shocks, whereas the second one is very weak under productivity shocks and significant if shocks to the pledgeability ratio are the driving force behind fluctuations.

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7 Appendix

7.1 The entrepreneur's consumption decision

Equations (8) and (9) in the text can be combined as follows,

$$\frac{p_t^h - \chi_t}{c_t^e} = \beta^e E_t \left\{ \frac{(1-\tau^e)\,\nu p_{t+1}^I y_{t+1}/h_t^e + p_{t+1}^h - R_t^e \chi_t/\pi_{t+1}}{c_{t+1}^e} \right\}, \tag{39}$$

where $\chi_t \equiv m_t E_t \pi_{t+1} p_{t+1}^h / R_t^e$. The latter definition allows us in turn to write the collateral constraint (equation 6) as

$$b_t = \chi_t h_t^e. \tag{40}$$

Define real net worth, nw_t, as the sum of after-tax real profits and beginning-of-period real estate wealth, minus real debt repayments,

$$\begin{aligned} nw_t &\equiv (1-\tau^e)\left(p_t^I y_t - w_t l_t^d\right) + p_t^h h_{t-1}^e - \frac{R_{t-1}^e}{\pi_t} b_{t-1} \\ &= (1-\tau^e)\,\nu p_t^I y_t + p_t^h h_{t-1}^e - \frac{R_{t-1}^e}{\pi_t}\chi_{t-1} h_{t-1}^e \\ &= \left[(1-\tau^e)\,\nu p_t^I y_t / h_{t-1}^e + p_t^h - \frac{R_{t-1}^e}{\pi_t}\chi_{t-1}\right] h_{t-1}^e, \end{aligned} \tag{41}$$

where in the first equality we have used (7) to substitute for $w_t l_t^d$ and (40) to substitute for b_{t-1}. We now guess that the entrepreneur consumes a fraction $1-\beta^e$ of her real net worth,

$$c_t^e = (1-\beta^e)\, nw_t. \tag{42}$$

Using (41) and (42) in equation (39), the latter collapses to

$$\frac{p_t^h - \chi_t}{c_t^e} = \frac{\beta^e}{1-\beta^e}\frac{1}{h_t^e}. \tag{43}$$

At the same time, the definition of real net worth and equation (40) allow us write the entrepreneur's budget constraint (equation 5) as

$$\chi_t h_t^e + nw_t = c_t^e + p_t^h h_t^e,$$

Combining the latter with equation (43), we finally obtain equation (42), which verifies our guess.

7.2 Implementation of the efficient steady state

Equations (1), (7) and (19) in the steady state jointly imply

$$c_{ss}\left(l_{ss}^{s}\right)^{\varphi}=\frac{1-\omega}{\omega}\left(1-\nu\right)p_{ss}^{I}\frac{y_{ss}^{s}}{l_{ss}^{s}}$$

The latter corresponds to its efficient counterpart (the steady state of equation 24) only if $p_{ss}^{I}=1$. In the steady state, equation (15) becomes $1+\tau=\left[\varepsilon/\left(\varepsilon-1\right)\right]p_{ss}^{I}$, where τ is the subsidy rate on the revenue of final goods producers. Therefore, steady-state efficiency requires setting the subsidy rate to

$$\tau=\frac{\varepsilon}{\varepsilon-1}-1.$$

On the other hand, the steady-state counterpart of equation (10), rescaled by y_{ss}, is given by

$$\frac{c_{ss}^{e}}{y_{ss}}=\left(1-\beta^{e}\right)\left[\left(1-\tau^{e}\right)\nu+\left(1-m\right)\frac{p_{ss}^{h}h_{ss}^{e}}{y_{ss}}\right], \tag{44}$$

where we have imposed $p_{ss}^{I}=1$ and we have also used the steady-state collateral constraint, $R_{ss}^{e}b_{ss}=mp_{ss}^{h}h_{ss}^{e}$. Similarly, the steady-state counterparts of equations (9) and (8) jointly imply

$$\frac{p_{ss}^{h}h_{ss}^{e}}{y_{ss}}=\beta^{e}\left[\left(1-\tau^{e}\right)\nu+\frac{p_{ss}^{h}h_{ss}^{e}}{y_{ss}}\right]+\left(\frac{1}{R_{ss}^{e}}-\beta^{e}\right)m\frac{p_{ss}^{h}h_{ss}^{e}}{y_{ss}},$$

which implies the following steady-state ratio of entrepreneurial real estate wealth over output,

$$\frac{p_{ss}^{h}h_{ss}^{e}}{y_{ss}}=\frac{\beta^{e}\left(1-\tau^{e}\right)\nu}{1-\beta^{e}-m\left(1/R_{ss}^{e}-\beta^{e}\right)}. \tag{45}$$

Using (45) to substitute for $p_{ss}^{h}h_{ss}^{e}/y_{ss}^{s}$ in (44), and imposing the steady-state efficiency requirement that $c_{ss}^{e}/y_{ss}=1-\omega$ (as a result of $c_{ss}=c_{ss}^{e}$ and $\left[1-\omega\right]y_{ss}=\omega c_{ss}+\left[1-\omega\right]c_{ss}^{e}$), we can solve for the tax rate that implements an efficient allocation in the steady state,

$$\tau^{e}=1-\frac{1-\omega}{\left(1-\beta^{e}\right)\nu}\frac{1-\beta^{e}-m\left(1/R_{ss}^{e}-\beta^{e}\right)}{1-m/R_{ss}^{e}}.$$

Finally, equation (3) implies that, in the steady state,

$$\frac{p_{ss}^{h}h_{ss}}{c_{ss}}=\frac{\vartheta}{1-\beta+\tau^{h}}.$$

Combining this with equation (45) and the efficiency requirement $c_{ss} = c^e_{ss} = (1-\omega)\, y_{ss}$, we have that the steady-state distribution of real estate in the decentralized economy is given by

$$\frac{h_{ss}}{h^e_{ss}} = \frac{(1-\omega)\,\vartheta}{\beta^e\,(1-\tau^e)\,\nu}\frac{1-\beta^e - m\,(1/R^e_{ss}-\beta^e)}{1-\beta+\tau^h}.$$

The latter coincides with the efficient steady-state distribution, $(1-\omega)\,\vartheta/\,(\beta\nu)$, only if

$$\tau^h = \frac{\beta}{\beta^e}\frac{1-\beta^e - m\,(1/R^e_{ss}-\beta^e)}{1-\tau^e} - (1-\beta)\,.$$

7.3 Derivation of the quadratic loss function

We start by performing a second order approximation (in logs) of the period utility function around the steady-state,

$$\begin{aligned} U_t &\equiv \omega\left[\log(c_t) + \vartheta_t \log(h_t) - \frac{(l^s_t)^{1+\varphi}}{1+\varphi}\right] + (1-\omega)\log(c^e_t) \\ &= \omega\hat{c}_t + (1-\omega)\,\hat{c}^e_t - \omega\,(l^s_{ss})^{1+\varphi}\left(\hat{l}_t + \frac{1+\varphi}{2}\hat{l}^2_t\right) + \omega\vartheta\left(\hat{h}_t + z^h_t\hat{h}_t\right) + t.i.p. + O^3, \quad (46)\end{aligned}$$

where hats denote log-deviations from steady state, the subscript ss indicates steady state values, $t.i.p.$ are terms independent of policy and O^3 collects all terms of order third and higher in the size of the shocks.

The aggregate resource constraint in goods markets, $(1-\omega)\,y_t/\Delta_t = \omega c_t + (1-\omega)\,c^e_t$, can be approximated by

$$(1-\omega)\left(\hat{y}_t + \frac{1}{2}\hat{y}^2_t - \hat{\Delta}_t\right) = \omega\frac{c_{ss}}{y_{ss}}\left(\hat{c}_t + \frac{1}{2}\hat{c}^2_t\right) + (1-\omega)\,\frac{c^e_{ss}}{y_{ss}}\left(\hat{c}^e_t + \frac{1}{2}\,(\hat{c}^e_t)^2\right) + O^3, \qquad (47)$$

where we have used the fact that $\hat{\Delta}_t$ is already a second-order term (see below). Equation (47) implies that

$$\hat{y}^2_t = \left(\frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\right)^2\hat{c}^2_t + \left(\frac{c^e_{ss}}{y_{ss}}\right)^2(\hat{c}^e_t)^2 + 2\frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\frac{c^e_{ss}}{y_{ss}}\hat{c}_t\hat{c}^e_t + O^3.$$

Using this to substitute for $\hat{y}_t^2$ in (47) and rearranging terms, we obtain

$$\begin{aligned}\hat{y}_t &= \frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\hat{c}_t + \frac{c_{ss}^e}{y_{ss}}\hat{c}_t^e + \hat{\Delta}_t + O^3 \\ &+\frac{1}{2}\left[\frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\left(1-\frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\right)\hat{c}_t^2 + \frac{c_{ss}^e}{y_{ss}}\left(1-\frac{c_{ss}^e}{y_{ss}}\right)\left(\hat{c}_t^e\right)^2 - 2\frac{\omega}{1-\omega}\frac{c_{ss}}{y_{ss}}\frac{c_{ss}^e}{y_{ss}}\hat{c}_t\hat{c}_t^e\right]. \quad (48)\end{aligned}$$

We now make use of our assumption of efficient steady state. This implies $c_{ss} = c_{ss}^e = (1-\omega)\, y_{ss}$. Using this in (48) yields

$$\hat{y}_t = \omega\hat{c}_t + (1-\omega)\,\hat{c}_t^e + \hat{\Delta}_t + \frac{\omega(1-\omega)}{2}\left(\hat{c}_t - \hat{c}_t^e\right)^2 + O^3. \quad (49)$$

The production function, $y_t = e^{a_t}\left[\omega l_t^s/(1-\omega)\right]^{1-\nu}\left(h_{t-1}^e\right)^{\nu}$, admits the following exact log-linear representation,

$$\hat{y}_t = a_t + (1-\nu)\,\hat{l}_t^s + \nu\hat{h}_{t-1}^e. \quad (50)$$

Using (49) and (50) to substitute for $\omega\hat{c}_t + (1-\omega)\,\hat{c}_t^e$ and $\hat{l}_t^s$ respectively in (46), we obtain

$$\begin{aligned}U_t &= \hat{y}_t - \hat{\Delta}_t - \omega\left(l_{ss}^s\right)^{1+\varphi}\left[\frac{\hat{y}_t - \nu\hat{h}_{t-1}^e}{1-\nu} + \frac{1+\varphi}{2}\left(\frac{\hat{y}_t - a_t - \nu\hat{h}_{t-1}^e}{1-\nu}\right)^2\right] \\ &-\frac{\omega(1-\omega)}{2}\left(\hat{c}_t - \hat{c}_t^e\right)^2 + \omega\vartheta\left(\hat{h}_t + z_t^h\hat{h}_t\right) + t.i.p. + O^3. \quad (51)\end{aligned}$$

In an efficient steady state, labor market equilibrium implies $c_{ss}\left(l_{ss}^s\right)^{\varphi} = (1-\nu)\, y_{ss}^s/\left(\frac{\omega}{1-\omega}l_{ss}^s\right)$, which combined with $c_{ss} = (1-\omega)\, y_{ss}$ implies $\omega\left(l_{ss}^s\right)^{1+\varphi} = 1-\nu$. Using this in (51), we have

$$U_t = \nu\hat{h}_{t-1}^e + \omega\vartheta\left(\hat{h}_t + z_t^h\hat{h}_t\right) - \frac{1+\varphi}{2(1-\nu)}\left(\hat{y}_t - \hat{y}_t^{*|h}\right)^2 - \frac{\omega(1-\omega)}{2}\left(\hat{c}_t - \hat{c}_t^e\right)^2 - \hat{\Delta}_t + t.i.p. + O^3, \quad (52)$$

where we have used the definition of efficient output (in log-deviations), $\hat{y}_t^{*|h} \equiv a_t + \nu\hat{h}_{t-1}^e$. Taking the present discount sum of (52), we have

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t U_t &= -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{1+\varphi}{1-\nu}\left(\hat{y}_t - \hat{y}_t^{*|h}\right)^2 + \omega(1-\omega)\left(\hat{c}_t - \hat{c}_t^e\right)^2\right] \\ &+\sum_{t=0}^{\infty}\beta^t\left[\beta\nu\hat{h}_t^e + \omega\vartheta\left(\hat{h}_t + z_t^h\hat{h}_t\right)\right] - \sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t + t.i.p. + O^3, \quad (53)\end{aligned}$$

where we have used the fact that $\hat{h}_{-1}^e$ and $\hat{\Delta}_{-1}$ are independent of policy as of time 0. The equilibrium condition in the real estate market, $\bar{h} = \omega h_t + (1-\omega) h_t^e$, can be approximated as follows,

$$\omega h_{ss} \left(\hat{h}_t + \frac{\hat{h}_t^2}{2} \right) + (1-\omega) h_{ss}^e \left(\hat{h}_t^e + \frac{\left(\hat{h}_t^e\right)^2}{2} \right) = O^3. \tag{54}$$

The latter equation implies that $\left(\hat{h}_t^e\right)^2 = [\omega/(1-\omega)]^2 (h_{ss}/h_{ss}^e)^2 \hat{h}_t^2 + O^3$. Using this and the efficient distribution of real estate in the steady state, $h_{ss}/h_{ss}^e = (1-\omega)\vartheta/(\beta\nu)$, equation (54) becomes

$$\omega\vartheta \hat{h}_t + \beta\nu \hat{h}_t^e = -\frac{\omega\vartheta}{2}\frac{\beta\nu + \omega\vartheta}{\beta\nu}\hat{h}_t^2 + O^3.$$

This implies

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \left[\beta\nu \hat{h}_t^e + \omega\vartheta \left(\hat{h}_t + z_t^h \hat{h}_t \right) \right] &= -\frac{\omega\vartheta}{2} \sum_{t=0}^{\infty} \beta^t \left(\frac{\beta\nu + \omega\vartheta}{\beta\nu} \hat{h}_t^2 - 2 z_t^h \hat{h}_t \right) + O^3 \\
&= -\frac{\omega\vartheta}{2} \frac{\beta\nu + \omega\vartheta}{\beta\nu} \sum_{t=0}^{\infty} \beta^t \left(\hat{h}_t - \hat{h}_t^* \right)^2 + t.i.p. + O^3, \quad (55)
\end{aligned}$$

where in the second equality we have used the definition of the efficient level of housing, $\hat{h}_t^* \equiv [\beta\nu/(\omega\vartheta + \beta\nu)] z_t^h$.

It is possible to show (see e.g. Woodford, 2003) that

$$\sum_{t=0}^{\infty} \beta^t \hat{\Delta}_t = \frac{\varepsilon}{2} \frac{\theta}{(1-\theta)(1-\beta\theta)} \sum_{t=0}^{\infty} \beta^t \hat{\pi}_t^2 + t.i.p. + O^3. \tag{56}$$

Using (55) and (56) in (53), we finally obtain

$$\sum_{t=0}^{\infty} \beta^t U_t = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O^3, \tag{57}$$

where

$$L_t = \frac{1+\varphi}{1-\nu} \left(\hat{y}_t - \hat{y}_t^{*|h} \right)^2 + \omega(1-\omega)(\hat{c}_t - \hat{c}_t^e)^2 + \omega\vartheta \frac{\beta\nu + \omega\vartheta}{\beta\nu} \left(\hat{h}_t - \hat{h}_t^* \right)^2 + \frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)} \hat{\pi}_t^2.$$

QED.

7.4 Log-linear equations

All variables in log-deviations from the efficient steady state. The log-linear constraints of the central bank's problem are the following.

1. Household's consumption Euler equation,

$$\hat{c}_t = E_t \hat{c}_{t+1} - E_t \left(\hat{R}_t^d - \hat{\pi}_{t+1} \right).$$

2. Household's demand for housing,

$$\left(1+\tau^h\right)\left(\hat{p}_t^h - \hat{c}_t\right) = \left(1+\tau^h-\beta\right)\left(z_t^h - \hat{h}_t\right) + \beta E_t \left(\hat{p}_{t+1}^h - \hat{c}_{t+1}\right).$$

3. Entrepreneur's borrowing constraint,

$$\hat{b}_t = z_t^m + E_t \hat{p}_{t+1}^h + \hat{h}_t^e - \left(\hat{R}_t^e - E_t \hat{\pi}_{t+1}\right).$$

4. Entrepreneur's consumption Euler equation,

$$\hat{c}_t^e = \beta^e R_{ss}^e E_t \left(\hat{c}_{t+1}^e - \hat{R}_t^e + \hat{\pi}_{t+1}\right) - \left(1-\beta^e R_{ss}^e\right)\hat{\xi}_t.$$

5. Entrepreneur's demand for real estate,

$$\begin{aligned} \hat{p}_t^h - \hat{c}_t^e &= \beta^e E_t \left\{ \frac{\left(1-\tau^e\right)\nu}{s_h^e}\left(\hat{y}_{t+1} + \hat{p}_{t+1}^I - \hat{h}_t^e\right) + \hat{p}_{t+1}^h - \left[\frac{\left(1-\tau^e\right)\nu}{s_h^e} + 1\right]\hat{c}_{t+1}^e \right\} \\ &\quad + m\left(\frac{1}{R_{ss}^e} - \beta^e\right)\left[z_t^m + \hat{\xi}_t + E_t \hat{p}_{t+1}^h - \left(\hat{R}_t^e - E_t \hat{\pi}_{t+1}\right)\right], \end{aligned}$$

   where $s_h^e \equiv p_{ss}^h h_{ss}^e / y_{ss}$.

6. Entrepreneur consumption,

$$\hat{c}_t^e = \frac{1-\beta^e}{1-\omega}\left[\left(1-\tau^e\right)\nu\left(\hat{y}_t + \hat{p}_t^I\right) + s_h^e\left(\hat{p}_t^h + \hat{h}_{t-1}^e\right) - s_h^e m\left(\hat{R}_{t-1}^e + \hat{b}_{t-1} - \hat{\pi}_t\right)\right].$$

7. Bank lending margin,

$$\hat{R}_t^e = \hat{R}_t^d$$
$$+\frac{\beta R_{ss}^e - 1}{\beta R_{ss}^e}\left[\frac{\hat{R}_t^d + \hat{p}_t^h - m\beta E_t\left(\hat{\pi}_{t+1} + \hat{p}_{t+1}^h + z_t^m\right)}{1 - m\beta} - \frac{\eta m\beta E_t\left(\hat{\pi}_{t+1} + \hat{p}_{t+1}^h + z_t^m\right) - \left(\hat{R}_t^d + \hat{p}_t^h\right)}{\eta m\beta - 1}\right].$$

8. New Keynesian Phillips curve,

$$\hat{\pi}_t = \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{p}_t^I + \beta E_t\hat{\pi}_{t+1}.$$

9. Real marginal costs,

$$\hat{p}_t^I = \hat{c}_t - \hat{y}_t + \frac{1+\varphi}{1-\nu}\left(\hat{y}_t - a_t - \nu\hat{h}_{t-1}^e\right).$$

10. Equilibrium in goods markets,

$$\hat{y}_t = \omega\hat{c}_t + (1-\omega)\,\hat{c}_t^e.$$

11. Equilibrium in the real estate market,

$$\hat{h}_t = -\frac{\beta\nu}{\omega\vartheta}\hat{h}_t^e.$$

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Figure 1: Impulse-responses to a negative productivity shock

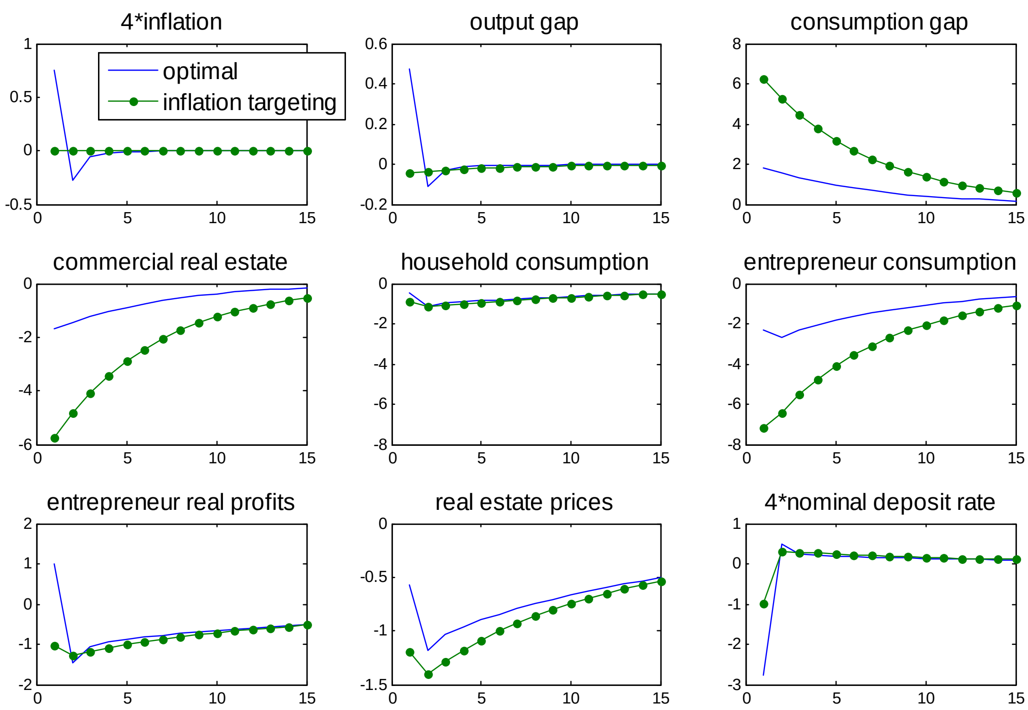

Figure 2: Impulse-responses to a negative credit-crunch shock